In this pdf we provide a complete proof that the trace and norm sheaf $\mathcal{O}$ and $\mathcal{O}^\times$ are presheaves with transfer in the sense of [MVW].
We will fix a base field k throughout the entire argument. All schemes or rings that appear are supposed to be separated of finite type or finitely generated over k, respectively.

The presheaf of global sections $\mathcal{O}$ and global invertible sections $\mathcal{O}^\times$ with transfer are defined by $\mathcal{O}(X) = \mathcal{O}_X(X)$, $\mathcal{O}^\times(X) = \mathcal{O}_X^\times(X)$. for $X \in Sm/k$.
For an elementary correspondence V from X to Y, which is an integral closed subscheme of $X \times Y$ finite surjective over X, we define $\mathcal{O}(V) : \mathcal{O}(X) \to \mathcal{O}(Y)$ and $\mathcal{O}^\times(V) : \mathcal{O}^\times(X) \to \mathcal{O}^\times(Y)$ by

$$\mathcal{O}(Y) \to \mathcal{O}(V) \xrightarrow{\mathrm{Tr}} \mathcal{O}(X),$$
$$\mathcal{O}^\times(Y) \to \mathcal{O}^\times(V) \xrightarrow{N} \mathcal{O}^\times(X),$$

respectively. Here Tr and N are induced by the usual trace and norm maps $\mathrm{Tr}_{K(V)/K(X)} : K(V) \to K(X)$ and $N_{K(V)/K(X)} : K(V) \to K(X)$ of function fields. Note that X is normal and $O(X)$ is integrally closed.
This definition extends to all finite correspondences $Cor(X,Y)$ by linearity.
We will show that this definition commutes with the composition of morphisms(finite correspondeces).

From now on we only treat the case of norm N; similar arguments apply to the trace. In order to prove that $O^\times$ commutes with the composition, we use the fact that if $L/K/F$ is a field tower, then $N_{L/F} = N_{K/F}N_{L/K}$. All the arguments and definition below are just technical ones.

Definition. *For a scheme X, let $X_1, \cdots, X_n$ be all the irreducible components of X. We define $K^\times(X) = \prod_i K^\times(X_i)$.*

Let η_i be the generic point of X_i, we define the local ring of X along X_i and the geometric multiplicity of X along X_i to be length of the artinian ring $\mathcal{O}_{X,X_i}$.

Definition. *For a finite surjective morphism $f : X \to Y$, let $X_1, \cdots, X_n$ be all the irreducible components of X. Then $Y_i = f(X_i)$ is an irreducible component of Y.*
Let $N_i : K^\times(X_i) \to K^\times(Y_i)$ be $N_i(\alpha) = N_{K(X_i)/K(Y_i)}(\alpha)^e$,
$e = l_{\mathcal{O}_{Y,Y_i}}(\mathcal{O}_{X,X_i})/[K(X_i) : K(Y_i)]$.
$l_A(M)$ means the length of M as an A-module.
Define $N_{X/Y} = N_f : K^\times(X) \to K^\times(Y)$ by $K^\times(X_i) \xrightarrow{N_i} K^\times(Y_i) \to K^\times(Y)$.

Remark. $N_{id_X} = id_{K^\times(X)}$.

Remark. $l_{\mathcal{O}_{Y,Y_i}}(\mathcal{O}_{X,X_i})/[K(X_i) : K(Y_i)]$ is an integer. First note that $\mathcal{O}_{X,X_i}$ is a component of finite scheme $X \times_Y \mathrm{Spec}\,\mathcal{O}_{Y,Y_i}$, so $\mathcal{O}_{X,X_i}$ is finite over $\mathcal{O}_{Y,Y_i}$.
Let $(A, \mathfrak{m}, K) \to (B, \mathfrak{n}, L)$ be a finite local homomorphism of artinian local rings.

Then we have

$$l_A(B) = l_A(B/\mathfrak{n}) + l_A(\mathfrak{n}/\mathfrak{n}^2) + \cdots$$

and

$$l_A(\mathfrak{n}^s/\mathfrak{n}^{s+1}) = l_K(\mathfrak{n}^s/\mathfrak{n}^{s+1}) = [L:K]\, l_L(\mathfrak{n}^s/\mathfrak{n}^{s+1}).$$

With this "extended" norm map, we have some convenient lemmas:

Lemma 1. *Let $V \subseteq X \times Y$ be a finite correspondence between smooth schemes X, Y, i.e., a closed subscheme finite surjective over X. Then the map $\mathcal{O}^\times(Y) \to \mathcal{O}^\times(V) \xrightarrow{N_{V/X}} \mathcal{O}^\times(X)$ is equal to the map $\mathcal{O}^\times([V]) : \mathcal{O}^\times(X) \to \mathcal{O}^\times(Y)$ defined by the finite correspondence $[V] = \sum n_i V_i$, where V_i are irreducible components of V and n_i are the geometric multiplicities of V along V_i.*

Lemma 2. *Let $f : X \to Y$ and $g : Y \to Z$ be finite surjective morphisms. Then $N_{gf} = N_g N_f$.*

(Proof). Let $\{Z_i\}_i$ be all the irreducible components of Z, $\{Y_{i,j}\}_j$ all the irreducible components of Y such that $g(Y_{i.j}) = Z_i$ and $\{X_{i,j,k}\}_k$ all the irreducible components of Z such that $f(X_{i,j,k}) = Y_{i,j}$.
$K^\times(Y) \xrightarrow{N_g} K^\times(Z)$ is given by $(\alpha_{i,j})_{i,j} \mapsto (\prod_j N_{K(Y_{i,j})/K(Z_i)}(\alpha_{i,j})^{e_{i,j}})_i$ and
$K^\times(X) \xrightarrow{N_f} K^\times(Y)$ is given by $(\alpha_{i,j,k})_{i,j,k} \mapsto (\prod_k N_{K(X_{i,j,k})/K(Y_{i,j})}(\alpha_{i,j,k})^{e_{i,j,k}})_{i,j}$
with $e_{i,j} = l_{\mathcal{O}_{Z,Z_i}}(\mathcal{O}_{Y,Y_{i,j}})/[K(Y_{i,j}) : K(Z_i)]$
and $e_{i,j,k} = l_{\mathcal{O}_{Y,Y_{i,j}}}(\mathcal{O}_{X,X_{i,j,k}})/[K(X_{i,j,k}) : K(Y_{i,j})]$.
By the fact that $N_{K(X_{i,j,k})/K(Y_{i,j})} N_{K(Y_{i,j})/K(Z_i)} = N_{K(X_{i,j,k})/K(Z_i)}$ and that $e_{i,j} e_{i,j,k} = l_{\mathcal{O}_{Z,Z_i}}(\mathcal{O}_{X,X_{i,j,k}})/[K(X_{i,j,k}) : K(Z_i)]$ we get the lemma.

□

For our purpose we need a "base change theorem". The preceding definition, however, is not suitable for this and we need an alternative definition of the norm map of schems.

Definition. *Let A be a ring and M be an A-module free of finite rank. For an A-endomorphism $\phi : M \to M$, we define $Norm_A(\phi) = \det(\phi)$.*
If $M = B$ is also an A-algebra, then for any $b \in B$, we define $Norm_A(b) = Norm_A(m_b)$ where $m_b : x \mapsto bx$.

With this definition, the "base change theorem" is easy to see:

Lemma 3. *Let B be an A-algebra, free of finite rank as an A-module and let $i : A \to A'$ be an A-algebra, and let $B' = B \otimes_A A'$. Then for any $b \in B$, $i(Norm_A(b)) = Norm_{A'}(b \otimes_A 1)$.*

Lemma 4. *Let A be an integral domain, B an A-algebra, free of finite rank as an A-module. Then $N_{B/A}(b) = Norm_A(b)$ for any $b \in B$.*

(Proof). Let $\mathfrak{q}_i$ $(i = 1, \cdots, n)$ be all the minimal prime ideals of B. By the going-down theorem, each $\mathfrak{q}_i$ lies above the prime ideal 0 of A.
Let $B_i = B_{\mathfrak{q}_i}$ and L_i be its residue field, and let K be the quotient field of A.
As $Norm$ commutes with base change and N is local at the generic points, by replacing A by K and B by $B \otimes_A K$, we may assume that $A = K$ and $B = B_1 \times \cdots \times B_n$.
Now we can furthermore reduce to the case where $(B, \mathfrak{q}, L)$ is an artinian local ring. Then the theorem is straightforward: consider $\mathfrak{q}^s/\mathfrak{q}^{s+1}$. The norm of the multiplication by b on $\mathfrak{q}^s/\mathfrak{q}^{s+1}$ is $N_{L/K}(b)^{\dim_L \mathfrak{q}^s/\mathfrak{q}^{s+1}}$; therefore we have

$$Norm_K(b) = N_{L/K}(b)^{\sum_s \dim_L \mathfrak{q}^s/\mathfrak{q}^{s+1}} = N_{B/A}(b)$$

since $\sum_s \dim_L \mathfrak{q}^s/\mathfrak{q}^{s+1} = l_K(B)/[L : K]$. □

Corollary 5. *Let $f : T \to S$ be a finite morphism to a normal scheme, $S' \to S$ any morphism from an integral scheme, and $f' : T' = T \times_S S' \to S'$ the base change of f. Then the diagram below is commutative:*

$$\begin{array}{ccc}
K^\times(T') & \xrightarrow{N} & K^\times(S') \\
\uparrow & & \uparrow \\
\mathcal{O}^\times(T) & \xrightarrow{N} & \mathcal{O}^\times(S)
\end{array}$$

Let X, Y, Z be smooth schemes, V and W be elementary correspondences between X, Y and Y, Z, respectively. Consider the diagram

$$\begin{array}{ccccccc}
\mathcal{O}^\times(Z) & \longrightarrow & \mathcal{O}^\times(V \times_Y Z) & & & & \\
\downarrow & & \downarrow & & & & \\
\mathcal{O}^\times(W) & \longrightarrow & \mathcal{O}^\times(V \times_Y W) & \longrightarrow & K^\times(V \times_Y W) & \xrightarrow{N} & K^\times(X \times_Y W) \\
{\scriptstyle N}\downarrow & & {\scriptstyle N}\downarrow & & {\scriptstyle N}\downarrow & & {\scriptstyle N}\downarrow \\
\mathcal{O}^\times(Y) & \longrightarrow & K^\times(V) & = & K^\times(V) & \xrightarrow{N} & K^\times(X).
\end{array}$$

The right-most square is commutative by Lemma 2. The left-most square on the row below is commutative by Lemma 3 and 4 (note that Y, V and W are integral).
Now the composition $\mathcal{O}^\times(Z) \to \mathcal{O}^\times(Y) \to \mathcal{O}^\times(X)$ defined using finite correspondences V and W is equal to the map $\mathcal{O}^\times(Z) \to \mathcal{O}^\times(V \times_Y W) \xrightarrow{N} \mathcal{O}^\times(X)$.
Let $p : X \times Y \times Z \to X \times Z$ be the projection. $V \times_Y W$ is a closed subscheme of $X \times Y \times Z$. The composition of V and W in $Cor(-,-)$ is defined to be $p_*([V \times_Y W]) \in Cor(X, Z)$. The push-forward is defined in the way similar to intersection theory; in this case every irreducible component of $V \times_Y W$ is finite surjective over X, see [MVW] for more detail.
Now , using Lemma 1, we are reduced to show the following lemma:

Lemma 6. *Let $p : C \to X \times Z$ a morphism from an integral scheme C whose composition with the projection $X \times Z \to X$ is finite surjective. Then the map $\mathcal{O}^\times(Z) \to \mathcal{O}^\times(C) \xrightarrow{N_{K(C)/K(X)}} \mathcal{O}^\times(X)$ coincides with the map $\mathcal{O}^\times(p_*C) : \mathcal{O}^\times(Z) \to \mathcal{O}^\times(X)$ defined by the finite correspondence $p_*C = dp(C)$, $d = [K(C) : K(p(C))]$.*

(Proof). For a $x \in K^\times(p(C))$ we have

$$N_{K(C)/K(X)}(x) = N_{K(D)/K(X)} N_{K(C)/K(D)}(x) = N_{K(D)/K(X)}(x)^d.$$

□

References

[MVW] Mazza, Carlo; Voevodsky, Vladimir; Weibel, Charles (2006), Lecture notes on motivic cohomology, Clay Mathematics Monographs, 2, Providence, R.I.: American Mathematical Society.